MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

DEM5038 – ENGINEERING MATHEMATICS 3

(Group E8)

3 MARCH 2017
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **11** pages (**4** pages with **5** questions and 7 pages for appendix).

2. Answer **ALL** questions. All necessary working steps must be shown.

3. Write all your answers in the answer booklet provided.

Please answer ALL questions and show the necessary working. Total mark is equal to 100.

Question 1

a) Find the general solution to the following non-homogenous differential equation. $y'''+y'=4\sin x+2x^2$ [15 marks]

b) Find the solution to the following differential equation by using **Laplace transform** method.

$$y''+7y'+10y=4te^{-3t}, \qquad y(0)=0, \quad y'(0)=-1$$ [12 marks]

[Total = 27 marks]

Question 2

a) The Figure 1 below is showing one period of a periodic function.

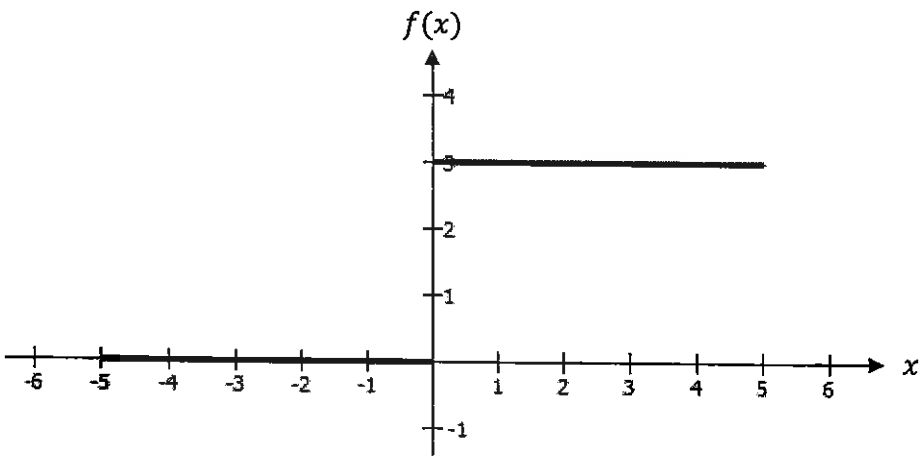


Figure 1

Find the Fourier series for the above function. [10 marks]

b) Find the Half-range **Cosine** series for

$$f(x)=\pi x-x^2, \qquad 0\le x\le \pi$$ [9 marks]

[Total = 19 marks]

Continued...

Question 3

a) Determine the Laplace transform of the following functions.

i. $e^{-4t}\cosh t$ [2 marks]

ii. $2t^4e^{6t} - 3t^2$ [2 marks]

iii. $\int_0^t 5\sin 2u\,du$ [2 marks]

iv. $f(t) = \begin{cases} t-5, & 0 < t < 5 \\ 5, & t > 5 \end{cases}$ [4 marks]

b) Find the **inverse** Laplace transform for the following functions.

i. $\dfrac{20}{(s+5)^6}$ [2 marks]

ii. $\dfrac{2}{(s-1)^2+9}$ [2 marks]

iii. $\dfrac{9s^2+12s}{s^4}$ [3 marks]

iv. $\dfrac{s+3}{s^2+9}$ [2 marks]

[Total = 19 marks]

Continued...

Question 4

a) Table 1 shows the preferred cat food by a sample of 50 cats with 3 categories; small cats (under 2 kg), medium cats (2 kg to 5 kg) and large cats (above 5 kg).

Table 1

Food	Small cats	Medium cats	Large cats
dry	2	7	4
canned	18	13	6

Find the probability that a randomly selected cat

i. is preferred canned food. [1 mark]

ii. is a large cat and preferred dry food. [2 marks]

iii. is preferred canned food, given that it is medium cat. [2 marks]

iv. is medium or small cat. [2 marks]

b) The manager of the women's dress department of a department store wants to know whether the true average number of women's dresses sold per day is 24. If a random sample of 36 days average number of dresses sold is 23 with a standard deviation of 7 dresses, at the 0.05 level of significance, is there sufficient evidence that the average number of women's dresses sold per day is not 24?

[8 marks]

[Total = 15 marks]

Continued...

Question 5

a) A study conducted at a certain college shows that 60% of the school graduates get their preferred job within a year after graduation. Using the binomial distribution function, find the probability that, within a year after graduation, among 14 randomly selected graduates of that college

i. at least 6 will get their preferred job. [2 marks]

ii. between 5 to 8 graduates will get their preferred job. [2 marks]

b) Assume that the actual amount of materials in containers is normally distributed with mean of 200 litres and a standard deviation of 15 litres. What is the probability that a container contains

i. between 191 to 209 litres? [2 marks]

ii. more than 224 litres? [2 marks]

c) A firm has six offices with various numbers of employees. Free coffee is provided for the employees. The manager wants to know if there is a relationship between the number of cups of coffee provided and the number of employees in the office. A sample of the data is provided in Table 2 as shown.

Table 2

Number of employees x	**Number of cups of coffee** y
11	18
13	36
15	40
18	50
21	58
24	74

i. Determine the correlation coefficient between the number of employees and the number of cups of coffee. [9 marks]

ii. Find the estimate regression line of y on x, $\hat{y} = a + bx$. [3 marks]

[Total = 20 marks]

End of Page.

Appendix

Quadratic formula $$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Annihilator for special functions

Annihilator	
D^n	$1, x, x^2, x^3, x^4, \ldots\ldots, x^{n-1}$
$(D-a)^n$	$e^{ax}, xe^{ax}, x^2e^{ax}, x^3e^{ax}, x^4e^{ax}, \ldots\ldots, x^{n-1}e^{ax}$
$\left(D^2 - 2aD + (a^2+b^2)\right)^n$	$e^{ax}\cos bx, xe^{ax}\cos bx, x^2e^{ax}\cos bx, x^3e^{ax}\cos bx, \ldots\ldots, x^{n-1}e^{ax}\cos bx$ and $e^{ax}\sin bx, xe^{ax}\sin bx, x^2e^{ax}\sin bx, x^3e^{ax}\sin bx, \ldots\ldots, x^{n-1}e^{ax}\sin bx$

Fourier Series

$$F(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{L}x + b_n \sin\frac{n\pi}{L}x\right)$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi}{L}xdx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi}{L}xdx$$

Integration Formula

$$\int \sin axdx = -\frac{1}{a}\cos ax$$

$$\int \cos axdx = \frac{1}{a}\sin ax$$

$$\int x\sin axdx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x\cos axdx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2\sin axdx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax)$$

$$\int x^2\cos axdx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax)$$

Linear Regression and Correlation

$$\hat{y} = a + bx$$

$$SS_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n} \qquad SS_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} \qquad SS_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$b = \frac{SS_{xy}}{SS_{xx}} \qquad a = \bar{y} - b\bar{x} = \frac{\sum y}{n} - b\frac{\sum x}{n}$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}}$$

Transformation of some functions

$f(t)$	$L\{f(t)\}$
1	$\frac{1}{s}$, s>0
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$ $n = 1,2,3,\ldots$
e^{at}	$\frac{1}{s-a}$, s>a
$t\,e^{at}$	$\frac{1}{(s-a)^2}$
y'	$sY(s)-y(0)$
y''	$s^2Y(s)-sy(0)-y'(0)$

$f(t)$	$L\{f(t)\}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
$(1)u(t-a)$	$\frac{e^{-sa}}{s}$
$(t-a)u(t-a)$	$\frac{e^{-sa}}{s^2}$

Second Shift Theorem

$$L\{f(t-a)u(t-a)\} = e^{-as}L\{f(t)\}$$
$$L\{f(t)u(t-a)\} = e^{-as}L\{f(t+a)\}$$

Binomial Probabilities $\binom{n}{x}p^x q^{n-x}$

Binomial Formulae when using Cambridge Statistical Table

Key Formulas (if $p \le 0.5$)

1. $P(X=r) = B(r) - B(r-1)$
2. $P(X \ge r) = 1 - B(r-1)$
3. $P(X > r) = 1 - B(r)$
4. $P(a \le X \le b) = B(b) - B(a-1)$
5. $P(X \le r) = B(r)$

Key Formulas (if $p > 0.5$)

1. $P(X=r) = P(Y=n-r) = B(n-r) - B(n-r-1)$
2. $P(X \ge r) = P(Y \le n-r) = B(n-r)$
3. $P(X \le r) = P(Y \ge n-r) = 1 - B(n-r-1)$
4. $P(a \le X \le b) = P(n-b \le Y \le n-a)$
   $= B(n-a) - B(n-b-1)$

Normal Distribution $Z = \frac{X-\mu}{\sigma}$

Hypothesis Testing $S_x = \frac{s}{\sqrt{n}}$ $Z = \frac{\bar{x}-\mu}{S_x}$

TABLE 1. THE BINOMIAL DISTRIBUTION FUNCTION

$n = 13$	$r = 0$	1	2	3	4	5	6	7	8	9	10
$p = 0.01$	0.8775	0.9928	0.9997								
.02	.7690	.9730	.9980	0.9999							
.03	.6730	.9436	.9938	.9995							
.04	.5882	.9068	.9865	.9986	0.9999						
0.05	0.5133	0.8646	0.9755	0.9969	0.9997						
.06	.4474	.8186	.9608	.9940	.9993	0.9999					
.07	.3893	.7702	.9422	.9897	.9987	.9999					
.08	.3383	.7206	.9201	.9837	.9976	.9997					
.09	.2935	.6707	.8946	.9758	.9959	.9995	0.9999				
0.10	0.2542	0.6213	0.8661	0.9658	0.9935	0.9991	0.9999				
.11	.2198	.5730	.8349	.9536	.9903	.9985	.9998				
.12	.1898	.5262	.8015	.9391	.9861	.9976	.9997				
.13	.1636	.4814	.7663	.9224	.9807	.9964	.9995	0.9999			
.14	.1408	.4386	.7296	.9033	.9740	.9947	.9992	.9999			
0.15	0.1209	0.3983	0.6920	0.8820	0.9658	0.9925	0.9987	0.9998			
.16	.1037	.3604	.6537	.8586	.9562	.9896	.9981	.9997			
.17	.0887	.3249	.6152	.8333	.9449	.9861	.9973	.9996			
.18	.0758	.2920	.5769	.8061	.9319	.9817	.9962	.9994	0.9999		
.19	.0646	.2616	.5389	.7774	.9173	.9763	.9948	.9991	.9999		
0.20	0.0550	0.2336	0.5017	0.7473	0.9009	0.9700	0.9930	0.9988	0.9998		
.21	.0467	.2080	.4653	.7161	.8827	.9625	.9907	.9983	.9998		
.22	.0396	.1846	.4301	.6839	.8629	.9538	.9880	.9976	.9996		
.23	.0334	.1633	.3961	.6511	.8415	.9438	.9846	.9968	.9995	0.9999	
.24	.0282	.1441	.3636	.6178	.8184	.9325	.9805	.9957	.9993	.9999	
0.25	0.0238	0.1267	0.3326	0.5843	0.7940	0.9198	0.9757	0.9944	0.9990	0.9999	
.26	.0200	.1111	.3032	.5507	.7681	.9056	.9701	.9927	.9987	.9998	
.27	.0167	.0971	.2755	.5174	.7411	.8901	.9635	.9907	.9982	.9997	
.28	.0140	.0846	.2495	.4845	.7130	.8730	.9560	.9882	.9976	.9996	
.29	.0117	.0735	.2251	.4522	.6840	.8545	.9473	.9853	.9969	.9995	0.9999
0.30	0.0097	0.0637	0.2025	0.4206	0.6543	0.8346	0.9376	0.9818	0.9960	0.9993	0.9999
.31	.0080	.0550	.1815	.3899	.6240	.8133	.9267	.9777	.9948	.9991	.9999
.32	.0066	.0473	.1621	.3602	.5933	.7907	.9146	.9729	.9935	.9988	.9999
.33	.0055	.0406	.1443	.3317	.5624	.7669	.9012	.9674	.9918	.9985	.9998
.34	.0045	.0347	.1280	.3043	.5314	.7419	.8865	.9610	.9898	.9980	.9997
0.35	0.0037	0.0296	0.1132	0.2783	0.5005	0.7159	0.8705	0.9538	0.9874	0.9975	0.9997
.36	.0030	.0251	.0997	.2536	.4699	.6889	.8532	.9456	.9846	.9968	.9995
.37	.0025	.0213	.0875	.2302	.4397	.6612	.8346	.9365	.9813	.9960	.9994
.38	.0020	.0179	.0765	.2083	.4101	.6327	.8147	.9262	.9775	.9949	.9992
.39	.0016	.0151	.0667	.1877	.3812	.6038	.7935	.9149	.9730	.9937	.9990
0.40	0.0013	0.0126	0.0579	0.1686	0.3530	0.5744	0.7712	0.9023	0.9679	0.9922	0.9987
.41	.0010	.0105	.0501	.1508	.3258	.5448	.7476	.8886	.9621	.9904	.9983
.42	.0008	.0088	.0431	.1344	.2997	.5151	.7230	.8736	.9554	.9883	.9979
.43	.0007	.0072	.0370	.1193	.2746	.4854	.6975	.8574	.9480	.9859	.9973
.44	.0005	.0060	.0316	.1055	.2507	.4559	.6710	.8400	.9395	.9830	.9967
0.45	0.0004	0.0049	0.0269	0.0929	0.2279	0.4268	0.6437	0.8212	0.9302	0.9797	0.9959
.46	.0003	.0040	.0228	.0815	.2065	.3981	.6158	.8012	.9197	.9758	.9949
.47	.0003	.0033	.0192	.0712	.1863	.3701	.5873	.7800	.9082	.9713	.9937
.48	.0002	.0026	.0162	.0619	.1674	.3427	.5585	.7576	.8955	.9662	.9923
.49	.0002	.0021	.0135	.0536	.1498	.3162	.5293	.7341	.8817	.9604	.9907
0.50	0.0001	0.0017	0.0112	0.0461	0.1334	0.2905	0.5000	0.7095	0.8666	0.9539	0.9888

See page 4 for explanation of the use of this table.

TABLE 1. THE BINOMIAL DISTRIBUTION FUNCTION

$n = 13$	$r = 11$	12
$p = 0.01$		
.02		
.03		
.04		
0.05		
.06		
.07		
.08		
.09		
0.10		
.11		
.12		
.13		
.14		
0.15		
.16		
.17		
.18		
.19		
0.20		
.21		
.22		
.23		
.24		
0.25		
.26		
.27		
.28		
.29		
0.30		
.31		
.32		
.33		
.34		
0.35		
.36		
.37	0.9999	
.38	.9999	
.39	.9999	
0.40	0.9999	
.41	.9998	
.42	.9998	
.43	.9997	
.44	.9996	
0.45	0.9995	
.46	.9993	
.47	.9991	0.9999
.48	.9989	.9999
.49	.9986	.9999
0.50	0.9983	0.9999

$n = 14$	$r = 0$	1	2	3	4	5	6	7
$p = 0.01$	0.8687	0.9916	0.9997					
.02	.7536	.9690	.9975	0.9999				
.03	.6528	.9355	.9923	.9994				
.04	.5647	.8941	.9833	.9981	0.9998			
0.05	0.4877	0.8470	0.9699	0.9958	0.9996			
.06	.4205	.7963	.9522	.9920	.9990	0.9999		
.07	.3620	.7436	.9302	.9864	.9980	.9998		
.08	.3112	.6900	.9042	.9786	.9965	.9996		
.09	.2670	.6368	.8745	.9685	.9941	.9992	0.9999	
0.10	0.2288	0.5846	0.8416	0.9559	0.9908	0.9985	0.9998	
.11	.1956	.5342	.8061	.9406	.9863	.9976	.9997	
.12	.1670	.4859	.7685	.9226	.9804	.9962	.9994	0.9999
.13	.1423	.4401	.7292	.9021	.9731	.9943	.9991	.9999
.14	.1211	.3969	.6889	.8790	.9641	.9918	.9985	.9998
0.15	0.1028	0.3567	0.6479	0.8535	0.9533	0.9885	0.9978	0.9997
.16	.0871	.3193	.6068	.8258	.9406	.9843	.9968	.9995
.17	.0736	.2848	.5659	.7962	.9259	.9791	.9954	.9992
.18	.0621	.2531	.5256	.7649	.9093	.9727	.9936	.9988
.19	.0523	.2242	.4862	.7321	.8907	.9651	.9913	.9983
0.20	0.0440	0.1979	0.4481	0.6982	0.8702	0.9561	0.9884	0.9976
.21	.0369	.1741	.4113	.6634	.8477	.9457	.9848	.9967
.22	.0309	.1527	.3761	.6281	.8235	.9338	.9804	.9955
.23	.0258	.1335	.3426	.5924	.7977	.9203	.9752	.9940
.24	.0214	.1163	.3109	.5568	.7703	.9051	.9690	.9921
0.25	0.0178	0.1010	0.2811	0.5213	0.7415	0.8883	0.9617	0.9897
.26	.0148	.0874	.2533	.4864	.7116	.8699	.9533	.9868
.27	.0122	.0754	.2273	.4521	.6807	.8498	.9437	.9833
.28	.0101	.0648	.2033	.4187	.6491	.8282	.9327	.9792
.29	.0083	.0556	.1812	.3863	.6168	.8051	.9204	.9743
0.30	0.0068	0.0475	0.1608	0.3552	0.5842	0.7805	0.9067	0.9685
.31	.0055	.0404	.1423	.3253	.5514	.7546	.8916	.9619
.32	.0045	.0343	.1254	.2968	.5187	.7276	.8750	.9542
.33	.0037	.0290	.1101	.2699	.4862	.6994	.8569	.9455
.34	.0030	.0244	.0963	.2444	.4542	.6703	.8374	.9357
0.35	0.0024	0.0205	0.0839	0.2205	0.4227	0.6405	0.8164	0.9247
.36	.0019	.0172	.0729	.1982	.3920	.6101	.7941	.9124
.37	.0016	.0143	.0630	.1774	.3622	.5792	.7704	.8988
.38	.0012	.0119	.0543	.1582	.3334	.5481	.7455	.8838
.39	.0010	.0098	.0466	.1405	.3057	.5169	.7195	.8675
0.40	0.0008	0.0081	0.0398	0.1243	0.2793	0.4859	0.6925	0.8499
.41	.0006	.0066	.0339	.1095	.2541	.4550	.6645	.8308
.42	.0005	.0054	.0287	.0961	.2303	.4246	.6357	.8104
.43	.0004	.0044	.0242	.0839	.2078	.3948	.6063	.7887
.44	.0003	.0036	.0203	.0730	.1868	.3656	.5764	.7656
0.45	0.0002	0.0029	0.0170	0.0632	0.1672	0.3373	0.5461	0.7414
.46	.0002	.0023	.0142	.0545	.1490	.3100	.5157	.7160
.47	.0001	.0019	.0117	.0468	.1322	.2837	.4852	.6895
.48	.0001	.0015	.0097	.0399	.1167	.2585	.4549	.6620
.49	.0001	.0012	.0079	.0339	.1026	.2346	.4249	.6337
0.50	0.0001	0.0009	0.0065	0.0287	0.0898	0.2120	0.3953	0.6047

See page 4 for explanation of the use of this table.

TABLE 1. THE BINOMIAL DISTRIBUTION FUNCTION

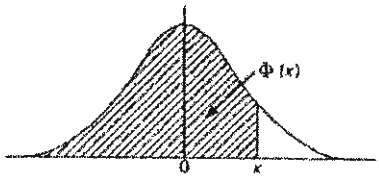
$n = 14$	$r = 8$	9	10	11	12	13
$p = 0.01$						
.02						
.03						
.04						
0.05						
.06						
.07						
.08						
.09						
0.10						
.11						
.12						
.13						
.14						
0.15						
.16	.9999					
.17	.9999					
.18	.9998					
.19	.9997					
0.20	0.9996					
.21	.9994	0.9999				
.22	.9992	.9999				
.23	.9989	.9998				
.24	.9984	.9998				
0.25	0.9978	0.9997				
.26	.9971	.9995	0.9999			
.27	.9962	.9993	.9999			
.28	.9950	.9991	.9999			
.29	.9935	.9988	.9998			
0.30	0.9917	0.9983	0.9998			
.31	.9895	.9978	.9997			
.32	.9869	.9971	.9995	0.9999		
.33	.9837	.9963	.9994	.9999		
.34	.9800	.9952	.9992	.9999		
0.35	0.9757	0.9940	0.9989	0.9999		
.36	.9706	.9924	.9986	.9998		
.37	.9647	.9905	.9981	.9997		
.38	.9580	.9883	.9976	.9997		
.39	.9503	.9856	.9969	.9995		
0.40	0.9417	0.9825	0.9961	0.9994	0.9999	
.41	.9320	.9788	.9951	.9992	.9999	
.42	.9211	.9745	.9939	.9990	.9999	
.43	.9090	.9696	.9924	.9987	.9999	
.44	.8957	.9639	.9907	.9983	.9998	
0.45	0.8811	0.9574	0.9886	0.9978	0.9997	
.46	.8652	.9500	.9861	.9973	.9997	
.47	.8480	.9417	.9832	.9966	.9996	
.48	.8293	.9323	.9798	.9958	.9994	
.49	.8094	.9218	.9759	.9947	.9993	
0.50	0.7880	0.9102	0.9713	0.9935	0.9991	0.9999

$n = 15$	$r = 0$	1	2	3
$p = 0.01$	0.8601	0.9904	0.9996	
.02	.7386	.9647	.9970	0.9998
.03	.6333	.9270	.9906	.9992
.04	.5421	.8809	.9797	.9976
0.05	0.4633	0.8290	0.9638	0.9945
.06	.3953	.7738	.9429	.9896
.07	.3367	.7168	.9171	.9825
.08	.2863	.6597	.8870	.9727
.09	.2430	.6035	.8531	.9601
0.10	0.2059	0.5490	0.8159	0.9444
.11	.1741	.4969	.7762	.9258
.12	.1470	.4476	.7346	.9041
.13	.1238	.4013	.6916	.8796
.14	.1041	.3583	.6480	.8524
0.15	0.0874	0.3186	0.6042	0.8227
.16	.0731	.2821	.5608	.7908
.17	.0611	.2489	.5181	.7571
.18	.0510	.2187	.4766	.7218
.19	.0424	.1915	.4365	.6854
0.20	0.0352	0.1671	0.3980	0.6482
.21	.0291	.1453	.3615	.6105
.22	.0241	.1259	.3269	.5726
.23	.0198	.1087	.2945	.5350
.24	.0163	.0935	.2642	.4978
0.25	0.0134	0.0802	0.2361	0.4613
.26	.0109	.0685	.2101	.4258
.27	.0089	.0583	.1863	.3914
.28	.0072	.0495	.1645	.3584
.29	.0059	.0419	.1447	.3268
0.30	0.0047	0.0353	0.1268	0.2969
.31	.0038	.0296	.1107	.2686
.32	.0031	.0248	.0962	.2420
.33	.0025	.0206	.0833	.2171
.34	.0020	.0171	.0719	.1940
0.35	0.0016	0.0142	0.0617	0.1727
.36	.0012	.0117	.0528	.1531
.37	.0010	.0096	.0450	.1351
.38	.0008	.0078	.0382	.1187
.39	.0006	.0064	.0322	.1039
0.40	0.0005	0.0052	0.0271	0.0905
.41	.0004	.0042	.0227	.0785
.42	.0003	.0034	.0189	.0678
.43	.0002	.0027	.0157	.0583
.44	.0002	.0021	.0130	.0498
0.45	0.0001	0.0017	0.0107	0.0424
.46	.0001	.0013	.0087	.0359
.47	.0001	.0010	.0071	.0303
.48	.0001	.0008	.0057	.0254
.49		.0006	.0046	.0212
0.50		0.0005	0.0037	0.0176

See page 4 for explanation of the use of this table.

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492 –	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506 –	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

3·075	0·9990	3·263	0·9994	3·731	0·99990	3·916	0·99995
3·105	0·9991	3·320	0·9995	3·759	0·99991	3·976	0·99996
3·138	0·9992	3·389	0·9996	3·791	0·99992	4·055	0·99997
3·174	0·9993	3·480	0·9997	3·826	0·99993	4·173	0·99998
3·215	0·9994	3·615	0·9998	3·867	0·99994	4·417	0·99999
			0·9999		0·99995		1·00000

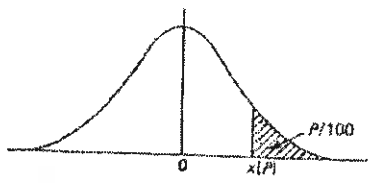
When $x \geqslant 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \dfrac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\dfrac{1}{x^2}+\dfrac{3}{x^4}-\dfrac{15}{x^6}+\dfrac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172